Esercizi 1.pdf

①

ⓐ $\log(n!) = \Theta(n \log n)$

$\exists n_0, e_1, e_2 > 0 : e_1 \, n \log n \le \log n! \le n \log n \cdot e_2 \quad \forall n \ge n_0$

$\log n! \ge e_1 \cdot n \log n \Leftrightarrow \log n! \ge e_1 \log n^n \quad \forall n \ge 1 \quad e_1 = 1$

$n \log n \cdot e_2 \ge \log n! \Leftrightarrow \log n^n \cdot e_2 \ge \log n! \quad \forall n \ge 1 \quad e_2 = 1$

ⓑ $\log(n!/2^n) = \Theta(n \log n)$

$\exists n_0, e_1, e_2 > 0 : e_1 \, n \log n \le \log(n!/2^n) \le e_2 \cdot n \log n$

$\log \dfrac{n!}{2^n} \ge e_1 \, n \log n \Leftrightarrow \log n! \cdot 2^{-n} \ge e_1 \, n \log n \Leftrightarrow$

$\Leftrightarrow \log n! + \log 2^{-n} \ge e_1 \, n \log n \Leftrightarrow \log n! + (-n) \ge e_1 \, n \log n$

$\log n! - n \ge e_1 \, n \log n$

Non esistono opportune costanti e_1, e_2 ed n_0 per provare tale relazione

ⓒ $n \log \sqrt{n} = \Theta(n \log n)$

$\exists n_0, e_1, e_2 > 0 : e_1 \, n \log n \le n \log \sqrt{n} \le e_2 \, n \log n$

$n \log \sqrt{n} \ge e_1 \, n \log n$

$n \log n^{1/2} \ge e_1 \, n \log n$

$\dfrac{1}{2} n \log n \ge e_1 \, n \log n \qquad\qquad \forall n \ge 1 \quad e_1 = \dfrac{1}{2}$

$e_2 \cdot n \log n \ge \dfrac{1}{2} n \log n \qquad\qquad \forall n \ge 1 \quad e_2 = \dfrac{1}{2}$