(8) Se $f_1(n) = \Theta(g_1(n))$ e $f_2(n) = \Theta(g_2(n))$ $\quad f_1(n) + f_2(n) = \Theta(g_1(n) + g_2(n))$

$\exists c_1, c_2, n_0 > 0: \quad c_1 n \leq f_1(n) \leq c_2 n \quad \forall n \geq n_0$

$\exists c_3, c_4, n_0 > 0: \quad c_3 n \leq f_2(n) \leq c_4 n \quad \forall n \geq n_0$

Se prendiamo un $c = c_1 + c_3$ e $c' = c_2 + c_4$ avremo

$\exists c, n_0 > 0: \quad f_1(n) + f_2(n) \geq c n \quad \forall n \geq 0$

$\quad ; \quad f_1(n) + f_2(n) \geq c_1 n + c_3 n$

$\quad ; \quad f_1(n) + f_2(n) \geq g_1(n) + g_2(n)$

$\exists c', n_0 > 0: \quad f_1(n) + f_2(n) \leq c' n \quad \forall n \geq 0$

$\quad ; \quad f_1(n) + f_2(n) \leq c_2 n + c_4 n$

$\quad : \quad f_1(n) + f_2(n) \leq g_1(n) + g_2(n) \quad n_0 = \max(n_0', n_0)$

E quindi abbiamo le soluzioni unendo i risultati.

(9) Se $f_1(n) = \Theta(g_1(n))$ e $f_2(n) = \Theta(g_2(n))$ $\quad f_1(n) + f_2(n) = \Theta(\max(g_1(n), g_2(n)))$

$\exists c_1, c_2, n_0 > 0: \quad c_1 g_1(n) \leq f_1(n) \leq c_2 g_1(n) \quad \forall n \geq n_0$

$\exists c_3, c_4, n_1 > 0: \quad c_3 g_2(n) \leq f_2(n) \leq c_4 g_2(n) \quad \forall n \geq n_1$

Se $g_1(n) = O(g_2(n))$

$\exists c_5, n_2 > 0: \quad g_1(n) \leq c_5 g_2(n) \quad \forall n \geq n_2$

~~Allora poniamo se $g_1(n) = \Omega(g_2(n))$ Allora~~

~~$\exists c_6, n_3 > 0: \quad g_1(n) \geq c_6 g_2(n) \quad \forall n \geq n_3$~~

~~Ponendo $n_2 = \max(n_2, n_3)$ abbiamo che~~

Allora, se poniamo $\quad n_2 = \max(n_0, n_1)$

$\exists n_2, c_x : c_x(g_1(n)) \leq f_1(n) + f_2(n) \leq c_x(g_1(n))$

Anche l'altro caso si verifica analogamente