

ES16

$$\sum_{x \in \text{left}} \text{key}[x] + \text{dr}(k) = \sum_{\substack{x \in \text{left} \\ x \leq k}} \text{key}[x] (\text{dr}_l(x)+1) + \sum_{\substack{x \in \text{right} \\ x > k}} \text{key}[x] (\text{dr}_r(x)+1)$$
$$= \sum_{x \in \text{left}} \text{key}[x] \text{dr}_l(x) + \sum_{x \in \text{left}} \text{key}[x] + \sum_{x \in \text{right}} \text{key}[x] \text{dr}_r(x) + \sum_{x \in \text{right}} \text{key}[x]$$

ALG016 (2)

if $z = \text{NULL}$
return 0

else
return $\text{ALG16}(z \rightarrow \text{left}) + \text{ALG15}(z \rightarrow \text{left}) + \text{ALG16}(z \rightarrow \text{right}) + \text{ALG15}(z \rightarrow \text{right})$

ALG015(z)

if $z = \text{NULL}$
return 0

else
 $a = \text{ALG015}(z \rightarrow \text{left})$
 $b = \text{ALG015}(z \rightarrow \text{right})$
if $a+b = 0$ return $\text{key}[z]$
else return $a+b$

ES16

ES18 applied to avl tree & 2 insertions



ES18

① ② ③

